

THREE FLAVOUR QUARK MATTER IN CHIRAL COLOUR DIELECTRIC MODEL

Sanjay K. Ghosh and S. C. Phatak

Institute of Physics, Bhubaneswar-751005, INDIA.

Abstract

We investigate the properties of quark matter at finite density and temperature using the nonlinear chiral extension of Colour Dielectric Model (CCM). Assuming that the square of the meson fields develop non- zero vacuum expectation value, the thermodynamic potential for interacting three flavour matter has been calculated. It is found that $\langle K^2 \rangle$ and $\langle \eta^2 \rangle$ remain zero in the medium whereas $\langle \pi^2 \rangle$ changes in the medium. As a result, u and d quark masses decrease monotonically as the temperature and density of the quark matter is increased. In the present model, the deconfinement density and temperature is found to be lower compared to lattice results. We also study the behaviour of pressure and energy density above critical temperature.

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I. INTRODUCTION

The quark structure of hadrons suggested the possibility of a transition from nuclear to quark phase at high density and/or temperature. This indicates that there can be a phase transition, usually termed as deconfinement transition, inside neutron stars at very high densities so that neutron stars may now consist of quark cores. On the other hand the heavy ion collision experiments can also produce quark matter. In fact the increasing availability of high energy beams is giving new dimension to the quest for quark gluon plasma. But the scenario gets complicated because of Chiral symmetry. At low density or temperature chiral symmetry is broken and eventually at some high density or temperature it is supposed to be restored. So with increase in density and/or temperature there can be two transitions, one is the deconfined transition where nucleons dissolve into quarks and the other is chiral transition where chiral symmetry is restored.

QCD is the theory of strong interaction. So one would like to understand the properties of matter at extreme conditions and the possible phase transitions using QCD. At present the only method of such investigation is lattice calculation. The best established results of lattice calculations [1] show that though the energy density attains an ideal gas value very quickly above deconfined transition temperature T_c , it takes a much larger value of T , for the pressure, to attain the same behaviour. This is an indication of nonperturbative behaviour which is poorly understood. The character of the phase transition depends sensitively on the number of light quark flavours. Although there are large uncertainties, the current status is that for 2-flavour light quarks, the transition is second order, while for 3 or more flavours it appears to be first order. But then lattice calculation has its own complicity, besides the problem of large computer time needed. Till today these are limited to systems with zero chemical potentials. It is unclear whether strange quarks should be treated as light or heavy. Also one should note that, in the real world since the mass of the light quarks are not strictly zero, the second order phase transition for light quark flavours may turn into a rapid cross over.

One can investigate the matter at extreme conditions, without so much of complicity, by using QCD motivated models, which have been developed and successfully applied to the study of the properties of baryons. One can get a simple physical understanding of the different characteristics of matter. There is one more advantage of using such models. In lattice, it is difficult to represent the chiral symmetry of the continuum. On the other hand, in models one can implement chiral symmetry exactly. Many such investigations have already been done [2- 6]. Recently, there are fresh attempts to understand the chiral symmetry transition using linear σ model [7, 8]. In ref. [7] authors have used $SU(3) \times SU(3)$ linear σ model Lagrangian in terms of the 3×3 matrix field M . In fact this Lagrangian corresponds to the action with M as order parameter. They find that for realistic meson masses, the chiral transition becomes a crossover phenomena, i.e the phase transition behave smoothly in all ther-

modynamic quantities. With the decrease in pion mass, the order of the transition changes from crossover to first order and with the decrease in σ mass, it changes from crossover to second order. Also at the transition point only the $SU(2) \times SU(2)$ part of the chiral symmetry is restored. Some authors have studied the temperature dependence of chiral condensate from an interacting pion gas, by using the fact that the condensate is related to the derivative of the free energy density with respect to the bare quark mass [9]. They find a critical temperature $T_c \sim \sqrt{2}f_\pi$.

In our earlier calculation [5], we have used nonlinear chiral colour dielectric model (CCDM) to study the properties of two flavour quark matter at finite temperature and density. There we have shown that with our ansatz of nonzero $F_\pi = \langle \vec{\pi}^2 \rangle / f_\pi^2$ and vanishing $\langle \vec{\pi} \rangle$, chiral symmetry is restored gradually at very high temperature $T \sim (2 - 3)T_c$, T_c being the deconfinement temperature which is around 100- 120 MeV. This model has been applied further to study the properties of stars [10, 11]. In the present paper we have extended our ansatz to study three flavour matter at finite density and temperature. Now our ansatz implies a nonzero $F_\phi = \langle \phi^2 \rangle / f_\pi^2$, where ϕ corresponds to the meson fields i.e. pion (π), kaon (K), or eta (η) [10- 12]. We find that in the medium F_K and F_η are zero where as F_π attains a nonzero value for the range of density and temperature we have considered. This implies that strange quark mass remains constant in the medium where as u and d quark masses decrease with density. So the chiral transition occur only in the two flavour sector in our model. Also the meson excitations play almost no role in determining the properties of quark matter.

The paper is organised as follows. In section 2 we define the CCDM and give a brief outline of our ansatz, most of the detail being left for appendix. In section 3, we present the results of our calculation and finally conclusion is given in section 4.

II. THE MODEL

The Colour Dielectric Model (CDM) is based on the idea of Nelson and Patkos [13]. In this model one generates the confinement of quarks and gluons dynamically through the interaction of these fields with scalar field or colour dielectric field χ . In the present work we have used the chiral extension [10, 11, 14] (CCDM) of this model to calculate quark matter EOS. The Lagrangian density of CCDM is given by

$$\begin{aligned}
L(x) = & \bar{\psi}(x)\{i\gamma^\mu\partial_\mu - (m_0 + m/\chi(x)U_5) + (1/2)g\gamma_\mu\lambda_a A_\mu^a(x)\}\psi \\
& + f_\pi^2/4Tr(\partial_\mu U\partial^\mu U^\dagger) - 1/2m_\phi^2\phi^2(x) - (1/4)\chi^4(x)(F_{\mu\nu}^a(x))^2 \\
& + (1/2)\sigma_v^2(\partial_\mu\chi(x))^2 - U(\chi)
\end{aligned} \tag{1}$$

where $U = e^{i\lambda_a\phi^a/f_\pi}$ and $U_5 = e^{i\lambda_a\phi^a\gamma_5/f_\pi}$, $\psi(x)$, $A_\mu(x)$, and $\chi(x)$ and $\phi(x)$ are quark, gluon, scalar (colour dielectric)and meson fields respectively. m and m_ϕ are the masses of quark and meson, f_π is the pion decay constant, $F_{\mu\nu}(x)$ is the usual colour

electromagnetic field tensor, g is the colour coupling constant and λ_a are the Gell-Mann matrices. The flavour symmetry breaking is incorporated in the Lagrangian through the quark mass term $(m_0 + m/\chi U_5)$, where $m_0 = 0$ for u and d quarks. So masses of u, d and s quarks are m , m and $m_0 + m$ respectively. So for a system with broken flavour symmetry i.e. flavour asymmetric matter, strange quark mass will be different from u and d quark masses. The meson matrix is then consist of a singlet η , triplet of π and quadruplet of K . One can also describe a flavour symmetric matter using above Lagrangian. For such systems, $m_0 = 0$ and all the quark masses become same. The meson matrix Φ then becomes a eight component field. The self interaction $U(\chi)$ of the scalar field is assumed to be of the form

$$U(\chi) = \alpha B \chi^2(x) [1 - 2(1 - 2/\alpha)\chi(x) + (1 - 3/\alpha)\chi^2(x)] \quad (2)$$

so that $U(\chi)$ has an absolute minimum at $\chi = 0$ and a secondary minimum at $\chi = 1$. The interaction of the scalar field with quark and gluon fields is such that quarks and gluons can not exist in the region where $\chi = 0$. In the limit of vanishing meson mass, the Lagrangian of eqn.(1) is invariant under chiral transformations of quark and meson fields.

There are two different approaches used when chiral models are applied for the study of hadrons. One is cloudy bag model [15], which is a perturbative method. The another one is hedgehog approach [16], which is nonperturbative but is not applicable for infinite matter. In our calculation, in an attempt to go beyond the perturbative approach of cloudy bag model, we assume that because of nonvanishing quark and antiquark densities, the square of the expectation value of meson fields develop a nonzero value i.e. $\langle \phi^2 \rangle \neq 0$. On the other hand we assume that expectation value of the meson field vanishes in the medium. For an infinite system of quarks we can assume that $\langle \phi^2 \rangle$ is independent of space and time. The meson excitations are then defined in terms of the fluctuations about $\langle \phi^2 \rangle$, so that $\phi^2 = \langle \phi^2 \rangle + \phi'^2$. Defining $F_\phi = \langle \phi^2 \rangle / f_\pi^2$, the CDM Lagrangian can be rewritten in terms of F_ϕ s and meson excitations ϕ' . In our calculation the scalar field χ and F_ϕ have been calculated in the mean field approximation and quark- gluon, gluon- gluon and quark- meson excitations are treated perturbatively. Thus, keeping the terms upto two meson exchange interaction we get,

$$\begin{aligned} L(x) = & \bar{\psi}(x) \{ i\gamma^\mu \partial_\mu - (m_0 + m/\chi(x)f_1(F_\phi)) + (1/2)g\gamma_\mu \lambda_a A_\mu^a(x) \} \psi \\ & - (1/4)\chi^4(x)(F_{\mu\nu}^a(x))^2 - U(\chi) + (1/2)\sigma_v^2(\partial_\mu \chi(x))^2 - \frac{1}{2}m_\phi^2 f_\pi^2 F_\phi \\ & + \frac{1}{2}f_2(F_\phi)(\partial_\mu(x))^2 - 1/2m_\phi^2 \phi'^2(x) - \frac{im}{f_\pi \chi} \bar{\psi}(x) \gamma_5 f_{I1}(F_\phi) \psi(x) \\ & + \frac{m}{2f_\pi \chi} \bar{\psi}(x) f_{I2} \psi(x) \end{aligned} \quad (3)$$

The last two terms are the interactions of meson excitations with quarks. The f_1 , and f_2 , are the 3×3 diagonal matrices, each term of which is a polynomial in F_ϕ s, i.e. F_π , F_K , F_η and their cross terms. f_{I1} and f_{I2} are interaction matrices for one

meson and two meson exchanges respectively. These are also 3×3 matrices but are not diagonal. In appendix , we have given the method of evaluating the f_1, f_2, f_{I1} and f_{I2} and also the expressions of their nonzero contributions for three flavour asymmetric quark matter, symmetric matter and two flavour quark matter. For three flavour (both flavour symmetric and asymmetric) the above polynomials are not summable, but are fast convergent and terms upto 3rd order gives reasonably good convergence. On the other hand, these polynomials are simpler for two flavour matter and can be summed to a compact form as given in appendix and ref. [5]. In our calculations scalar field χ and F_ϕ have been evaluated in the mean field approximation and quark- gluon and quark- meson interactions are treated perturbatively. In this approximation, χ is independent of space and time for uniform quark matter. In actual calculation, the mean field values of χ and F_ϕ are determined by minimizing the thermodynamical potential Ω . Here, to have a qualitative feeling of the different F_ϕ 's, let us consider the equation of motion for F_ϕ at zero temperature and without interactions.

$$(i\gamma^\mu \partial_\mu - (m_0 + \frac{m}{\chi} f_1(F_\phi))\psi(x) = 0 \quad (4)$$

$$U'(\chi) = \frac{m}{\chi^2} f_1(F_\phi) \frac{\gamma}{2\pi^2} \int_0^{k_f} k^2 dk m^* / (k^2 + m^{*2})^{1/2} \quad (5)$$

$$f_2(F_\phi) \partial^\mu \partial_\mu \phi'(x) + m_\phi^2 \phi'(x) = 0. \quad (6)$$

$$m_\phi^2 f_\pi^2 = \frac{m}{\chi} f_1'(F_\phi) \frac{\gamma}{2\pi^2} \int_0^{k_f} k^2 dk m^* / (k^2 + m^{*2})^{1/2} \quad (7)$$

where, $m^* = (m/\chi)(f_1(F_\phi))_{11or22}$ for u, d quarks and $m^* = m_0 + (m/\chi)(f_1(F_\phi))_{33}$ for s quarks. k_f is the fermi momentum and γ is the spin- colour degeneracy factor for quarks. The equation of motion for quarks (eqn.(4)) shows that m^* can be identified with the effective quark masses in the presence of χ and F . Also the equation of meson fluctuations show that, for non zero F the energy momentum relation for meson fluctuation is $E_\phi(k) = \sqrt{k^2 + m_\phi^{*2}}$, where $m_\phi^* = \frac{m_\phi}{\sqrt{f_2(F_\phi)}}$. Thus the effective pion mass increases with F and hence density in the medium. The mean field values of χ and F can be evaluated by solving the equations (4) - (7) self consistently.

The characteristics of F_ϕ can be understood from eqn.(7). The physical values of F_ϕ should be greater than zero. The eqn.(7) yields a positive value of F_ϕ only if $\frac{\gamma}{2\pi^2} \int_0^{k_f} k^2 dk m^* / (k^2 + m^{*2})^{1/2} \geq (m_\phi f_\pi \chi / m)^2$, that is if k_f (or the quark density) is larger than a certain value. Below this density, F_ϕ remains zero and, therefore, the eqn. (7) becomes redundant. Now since $m_\pi < m_K < m_\eta$, the respective fermi momentum k_f (for π) $< k_f$ (for K) $< k_f$ (for η), i.e. as quark density increases, the F_π will appear first. The F_K and F_η will appear much later as the masses of K and

η are much higher than the pion mass. Also, F_ϕ increases with quark density. This means that effective quark masses will decrease with density and may vanish for a certain value of F_ϕ .

Using the modified Lagrangian we calculate thermodynamic potential Ω upto second order in quark- gluon coupling and quark- meson couplings [4, 5]. The thermodynamic quantities are obtained from Ω using standard methods [4, 5]. The mean field values of χ and F_ϕ are obtained by minimizing Ω with respect to χ and F_ϕ ;

$$(\partial\Omega/\partial\chi)_{T,\mu} = 0 ; \quad (\partial\Omega/\partial F_\phi)_{T,\mu} = 0 \quad (8)$$

III. RESULTS AND DISCUSSION

The thermodynamic properties of the quark matter are calculated for a number of parameter sets of chiral CDM (B , m , α and g_s) which reproduce baryon masses. Earlier calculations show that these parameters are not determined uniquely by the fitting procedure [14]. In particular it has been found that good fits to baryon masses are obtained for $0.6GeV \leq m_{GB} \leq 3GeV$, $m_q(u, d) \leq 125MeV$, $m_q(s) \sim 300MeV$ and $B^{1/4} \leq 150MeV$, where $m_{GB} = \sqrt{2B\alpha/\sigma_v^2}$. On the other hand, if one insist for good fits of charge radius and magnetic moments as well, then the fits are better for lower values of m_{GB} , $m_q(u, d)$ and $B^{1/4}$. But, lower bag pressure $B^{1/4}$ gives a lower value for transition density and temperature. In fact in ref. [11] we found that for such parameter sets one even gets a stable quark matter phase at nuclear matter densities. So we have used parameter sets for which one gets reasonable behaviour and we find that the results for these parameter sets are qualitatively similar. Therefore we have considered a representative parameter set ($B^{1/4} = 152MeV$, $m_{u,d} = 92MeV$, $m_s = 295MeV$, $\alpha = 36$ and $g = 1.002$) for a detailed discussion. Since the analytic derivatives of the thermodynamic potential with respect to χ and F_ϕ s are nor easy to obtain, we have numerically minimized $\Omega(T, \mu)$ as function of χ and F_ϕ . We find that the mean field values of χ remain close to 1 for both two and three flavour matter, where as, for three flavour quark matter F_K and F_η remain zero in the medium for the range of density as well as temperature considered in our calculation. F_π is zero for small values of T and μ to about 2 or more for $T = 400MeV$ and $\mu = 500MeV$. From the expressions given in appendix it can be found that u and d quark couple with all the F_ϕ s' i.e. F_π , F_K and F_η . On the other hand, s quark couples with only K and η . So $F_K = F_\eta = 0$ implies that strange sector of the $SU(3) \times SU(3)$ decouples and strange quark mass remains constant in the medium. m_u and m_d become small at large values of baryon density n_B and temperature T . So, in our model, chiral symmetry restoration occurs for $SU(2) \times SU(2)$ sector only. The variation of F_π with n_B and T are shown in Fig.1 (a and b) and Fig.2(a and b) respectively. Corresponding curves for flavour symmetric matter are shown by the curves c and d. We find that variation of quark mass with n_B and T is slower for flavour symmetric matter.

The equation of motion for F_π [eqn.(7)] shows that for lower values of m_π , the density at which F_π is nonzero comes down. It is also obvious that one can not lower m_π arbitrarily as below a certain value of m_π , there is no solution. In Fig.3, we have plotted quark mass with n_B for different m_π . We find that for lower m_π , quark mass approaches zero faster i.e. the chiral restoration occur at lower density for lower pion mass. Similar behaviour is obtained for finite T and zero μ as well. But the nature of the chiral transition, in our calculation, does not change with m_π and chiral restoration phase is reached smoothly. This behaviour is different compared to ref. [7], where authors find that nature of transition changes from crossover to first order with decreasing pion mass. In our model, the effective pion mass increases with increase in density and temperature. The temperature variation of m_π is shown in Fig.4 It shows that initially pion mass increases to about $150MeV$ and then remain constant. As discussed earlier, as the chiral behaviour of the $SU(3) \times SU(3)$ is described by $SU(2) \times SU(2)$ part, the similar behaviour is observed for two flavour matter as well [5].

Figure 5 shows the variation of pressure with density for different values of temperature. At zero temperature the pressure is negative for $n_B < 0.35fm^3$. This density defines the boundary of the quark phase [5]. The density at which the pressure vanishes decreases with increasing temperature and above a certain critical temperature ($\sim 110 - 120MeV$ in our model) the pressure is always positive. This temperature can be defined as the critical temperature for the QGP phase transition in our model. We find that critical temperature for two flavour matter ($\sim 115MeV$) is higher than the three flavour matter ($\sim 110MeV$). For flavour symmetric case the transition temperature is lowest ($\sim 105MeV$). But, as evident from the values quoted, the T_c is not much sensitive to the different composition of the quark matter system. The best evidence from the lattice QCD calculations show that for two dynamical flavours transition temperature is around $150MeV$ and the transition density is around 4- 8 times the nuclear matter density. For three flavour matter there are no convincing calculations, but one can assume it to be not much less than $150MeV$, keeping in mind the large value of m_s compared to u and d quarks. So, in our model, critical temperature for deconfined transition is lower than the lattice QCD results.

For an ideal gas of quarks and gluons, $E/T^4 = \pi^2/30(\gamma_g + 7/4\gamma_q = 15.63$ for three flavour matter and $P = \frac{1}{3}E$. Lattice calculations [1] show that energy density increases above the T_c and quickly attains the ideal gas value. On the other hand pressure needs a larger temperature ($T \sim T_c$) to reach the ideal gas value. This is one indication that nonperturbative effects are still present, in the QGP, atleast just above the critical temperature. The values of E/T^4 and $3P/T^4$ for $\mu = 0$, from our model calculation, are given in Fig.6. The figure shows that at high temperature, E/T^4 approaches a value close to 15 and as the temperature is lowered, there is a small increase in E/T^4 . We do not show the results below the T_c as the calculation is not applicable at these temperatures. Fig.6 also shows that P/T^4 is a monotonically increasing function of temperature. We have also fitted E/T^4 and P/T^4 with

functions of the form $f(T) = \sum a_i/T^i$ with i running from 0 to 4. We find that a good fit to E/T^4 is obtained with $a_0 = 14.38$, $a_1 = 644.15$, $a_2 = -1.15 \times 10^5$, $a_3 = 0$ and $a_4 = 9.22 \times 10^8$. For P/T^4 , a good fit is obtained with $a_0 = 5.68$, $a_1 = -292.12$, $a_2 = 27753.10$, $a_3 = 0$ and $a_4 = -7.699 \times 10^8$. The corresponding fits for two flavour matter is similar to the one obtained in ref. [5]. Because of the constant background of the scalar field χ and F , we expect that the energy density and pressure should have a temperature independent term.

IV. CONCLUSION

The non-linear version of the chiral colour dielectric model is used to study the properties of quark matter in the present work. We have assumed that due to non-zero quark and antiquark densities, the square of the meson fields (pion, kaon and eta) develop a vacuum expectation value. In the matter, the $\langle K^2 \rangle$ and $\langle \eta^2 \rangle$ remain zero throughout the range of densities and temperatures considered. On the other hand, $\langle \pi^2 \rangle$ has a non-zero value in the medium. As a result, the s quark mass remain constant in the medium, where as, effective u and d quark masses decrease continuously as the density and temperature of the quark matter is increased. If we interpret this as a restoration of chiral symmetry, our calculation implies that chiral symmetry is restored smoothly in chiral colour dielectric model. Also, the chiral restoration occurs in $SU(2) \times SU(2)$ part only. There is a small increase in the pion mass with temperature and density in our model. Furthermore, if the pion mass is decreased, the quark mass drops faster i.e. the chiral restoration occurs for lower density and temperatures. Here one thing should be noted that quark- meson interactions play almost no role in determining the quark matter characteristics as their contribution to Ω is much less compared to pion background field and quark-gluon interaction contribution [5].

The pressure of the quark matter becomes negative at lower densities and temperatures, which implies that a confined phase is reached when the temperature or density of the quark matter is lowered. We find that the critical temperature does not vary much for different composition of the quark matter. The values of the critical temperature and density at which deconfinement transition take place are smaller than the values obtained in lattice calculations [1]. The behaviour of the pressure and density calculated for $\mu = 0$ is quantitatively similar to that obtained in lattice calculations.

APPENDIX A:

Here we will describe the method of calculating the Lagrangian (3) in terms of the $\frac{\langle \phi^2 \rangle}{f_\pi^2} = F_\phi$ and the meson excitations ϕ' . The meson octet Φ is,

$$\lambda_a \phi^a = \Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix} \quad (\text{A1})$$

where λ_a are the Gellmann matrices. The chiral symmetry operator and meson kinetic energy operator are $U_5 = e^{i\gamma_5 \Phi / f_\pi}$ and $U = e^{i\Phi / f_\pi}$. General procedure is to expand the exponential and contract the different fields following the different rules as prescribed below.

$$\pi^+ \pi^- = \pi^- \pi^+ = \pi^{02} = \langle \pi^2 \rangle / j_\pi = F_\pi f_\pi^2 / j_\pi \quad (\text{A2})$$

$$K^+ K^- = K^- K^+ = K^0 \bar{K}^0 = \bar{K}^0 K^0 = \langle K^2 \rangle / j_K = F_K f_\pi^2 / j_K \quad (\text{A3})$$

$$\eta^2 = \langle \eta^2 \rangle / j_\eta = F_\eta f_\pi^2 / j_\eta \quad (\text{A4})$$

where in a flavour asymmetric matter the factor due to isospin recoupling are $j_\pi = 3$ for pions, $j_K = 4$ for kaons and $j_\eta = 1$ for eta. For flavour symmetric matter $j_\pi = j_K = j_\eta = 8$.

In general for a operator $O(\phi(x))$, for any +ve integer n , we can write,

$$O(\phi(x)) = \langle O(\phi(x)) \rangle + \sum_n [O(\phi(x))]_n \quad (\text{A5})$$

where $[O(\phi(x))]_n$ implies that there are n meson fields ($\phi'(x)$) in the expression. For our present calculation, we need terms upto $n = 2$. Let us now evaluate the expansion for different combinations of meson fields.

$$\begin{aligned} \langle (\pi^+ \pi^-)^n \rangle &= n \langle \pi^+ \pi^- \rangle \langle (\pi^+ \pi^-)^{n-1} \rangle \\ &= n(n-1) \langle \pi^+ \pi^- \rangle^2 \langle (\pi^+ \pi^-)^{n-2} \rangle \\ &= \frac{n!}{j_\pi^n} \langle \pi^2 \rangle^n \end{aligned} \quad (\text{A6})$$

where the factor n implies that there are n ways to contract one $(\pi^+ \pi^-)$ out of n $(\pi^+ \pi^-)$ fields. Once first contraction is done, then second one has $n-1$ possibilities. Hence we get the factor $n(n-1)$. Similarly one can get the form for other fields.

$$\begin{aligned} \langle (\pi^0)^{2n} \rangle &= (2n-1) \langle \pi^{02} \rangle \langle (\pi^0)^{2n-2} \rangle \\ &= (2n-1)(2n-3) \langle \pi^0 \rangle^2 \langle (\pi^0)^{2n-4} \rangle \end{aligned} \quad (\text{A7})$$

$$= \frac{(2n-1)!!}{j_\pi^n} \langle \pi^2 \rangle^n \quad (\text{A8})$$

$$\langle (\eta)^{2n} \rangle = (2n-1)!! j_\eta^n \langle \eta \rangle^n \quad (\text{A9})$$

$$\langle (K^+ K^-)^n \rangle = \frac{n!}{j_K^n} \langle K^2 \rangle^n \quad (\text{A10})$$

$$\langle (K^0 \bar{K}^0)^n \rangle = \frac{n!}{j_K^n} \langle K^2 \rangle^n \quad (\text{A11})$$

The two meson interaction term can be evaluated following the above procedure.

$$[(\pi^+ \pi^-)^n]_2 = \frac{n^2(n-1)!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^+ \pi^- \quad (\text{A12})$$

The n^2 factor shows that there are $n \times n$ ways to collect a $\pi^+ \pi^-$ pair from n pairs. The remaining $n-1$ pairs can be contracted in $(n-1)!$ ways. For other meson fields we get,

$$[(\pi^0)^{2n}]_2 = \frac{n(2n-1)(2n-3)!!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^0 \pi^0 \quad (\text{A13})$$

$$[\eta^{2n}]_2 = \frac{n(2n-1)(2n-3)!!}{j_\eta^{n-1}} \langle \eta^2 \rangle^{n-1} \eta^2 \quad (\text{A14})$$

$$[(K^+ K^-)^n]_2 = \frac{n^2(n-1)!}{j_K^{n-1}} \langle K^2 \rangle^{n-1} K^+ K^- \quad (\text{A15})$$

$$[(K^0 \bar{K}^0)^n]_2 = \frac{n^2(n-1)!}{j_K^{n-1}} \langle K^2 \rangle^{n-1} K^0 \bar{K}^0 \quad (\text{A16})$$

For odd m , $[\phi^{2n}]_m$ vanishes, since $\langle \phi \rangle = 0$. Also for odd powers, $\langle \phi \phi^2 \rangle = 0$. The one meson exchange terms are calculated from the terms containing odd powers of the meson concerned. For any +ve integer n we can write,

$$\begin{aligned} [(\pi^+)^n (\pi^-)^{n+1}]_1 &= (n+1) \pi^- \langle \pi^+ \pi^- \rangle^n \\ &= \frac{(n+1)n!}{j_\pi^n} \langle \pi^2 \rangle^n \pi^- \end{aligned} \quad (\text{A17})$$

where one π^- can be picked up from the $n+1$ fields in $n+1$ ways and rest of the n pairs of $\pi^+ \pi^-$ can be contracted in $n!$ ways. Similarly,

$$[(\pi^+)^{n+1} (\pi^-)^n]_1 = \frac{(n+1)n!}{j_\pi^n} \langle \pi^2 \rangle^n \pi^+ \quad (\text{A18})$$

$$[(\pi^0)^{2n-1}]_1 = \frac{(2n-1)(2n-3)!!}{j_\pi^{n-1}} \langle \pi^2 \rangle^{n-1} \pi^0 \quad (\text{A19})$$

$$[\eta^{2n-1}]_1 = \frac{(2n-1)(2n-3)!!}{j_\eta^{n-1}} \langle \eta^2 \rangle^{n-1} \eta \quad (\text{A20})$$

$$[(K^+)^n(K^-)^{n+1}]_1 = = \frac{(n+1)n!}{j_K^n} < K^2 >^n K^- \quad (A21)$$

$$[(K^+)^{n+1}(K^-)^n]_1 = = \frac{(n+1)n!}{j_K^n} < K^2 >^n K^+ \quad (A22)$$

$$[(K^0)^n(\bar{K}^0)^{n+1}]_1 = = \frac{(n+1)n!}{j_K^n} < K^2 >^n \bar{K}^0 \quad (A23)$$

$$[(K^0)^{n+1}(\bar{K}^0)^n]_1 = = \frac{(n+1)n!}{j_K^n} < K^2 >^n K^0 \quad (A24)$$

$$(A25)$$

Case a: Three flavour asymmetric matter

So starting from U_5 , considering only the even powers we get a 3×3 diagonal matrix $(f_1(F_\phi))_{ii} = < U_5 >$, with fully contracted terms, which contributes to the quark mass terms.

$$\begin{aligned} (f_1(F_\phi))_{11} = (f_1(F_\phi))_{22} = & 1 - \frac{F_\pi}{2!} + \frac{5F_\pi^2}{3 \cdot 4!} - \frac{35F_\pi^3}{9 \cdot 6!} \dots \\ & - \frac{F_k}{2 \cdot 2!} + \frac{3F_k^2}{4 \cdot 4!} - \frac{3F_k^3}{2 \cdot 6!} \dots \\ & - \frac{F_\eta}{3 \cdot 2!} + \frac{F_\eta^2}{3 \cdot 4!} - \frac{5F_\eta^3}{9 \cdot 6!} \dots \\ & + F_\eta \left(\frac{2F_\pi}{4!} - \frac{25F_\pi^2}{3 \cdot 6!} + \dots \right) \\ & - 5 F_\eta^2 \frac{F_\pi}{5!} \dots + F_k \left(\frac{3F_\pi}{2 \cdot 4!} - \frac{25F_\pi^2}{6 \cdot 6!} + \dots \right) \\ & - 7 F_k^2 \frac{F_\pi}{2 \cdot 6!} \dots + F_\eta \left(\frac{F_k}{2 \cdot 4!} - \frac{3 F_k^2}{2 \cdot 6!} \dots \right) - 3 F_\eta^2 \frac{F_k}{2 \cdot 6!} \dots + O(F_\phi^4) \end{aligned} \quad (A26)$$

$$\begin{aligned} (f_1(F_\phi))_{33} = & 1 - \frac{F_k}{2 \cdot 2!} + \frac{3F_k^2}{2 \cdot 4!} - \frac{3 F_k^3}{6!} \dots \\ & - \frac{4 F_\eta}{3 \cdot 2!} + \frac{16 F_\eta^2}{3 \cdot 4!} - \frac{320 F_\eta^3}{9 \cdot 6!} \dots \\ & + F_\eta \left(\frac{2 F_\pi}{4!} - \frac{25 F_\pi^2}{3 \cdot 6!} \right) \dots \\ & - F_\eta^2 \frac{5 F_\pi}{6!} \dots + F_k \left(\frac{F_\pi}{4!} - \frac{5 F_\pi^2}{3 \cdot 6!} \right) \dots \\ & - F_k^2 \frac{7 F_\pi}{2 \cdot 6!} \dots + F_\eta \left(\frac{3 F_k}{4!} - \frac{15 F_k^2}{2 \cdot 6!} \right) \dots \\ & - 19 F_\eta^2 \frac{F_k}{6!} - \frac{2 F_\pi F_k F_\eta}{6!} \dots \\ & + O(F_\phi^4) \end{aligned} \quad (A27)$$

As stated in the text, we find that $F_k = F_\eta = 0$ for the range of density ($\sim 15 - 20$ times normal nuclear matter density) and temperature ($\sim 3 - 4$

times the critical temperature) considered. This implies that in the above expression, the terms with only F_π will contribute. The resultant terms can be summed up to give the new masses of u , d and s quarks. So, for flavour asymmetric quark matter the nonzero corrections to quark mass (u and d) terms are

$$(f_1(F_\pi))_{11} = (f_1(F_\pi))_{22} = (1 - F_\pi)e^{-F_\pi/6} \quad (\text{A28})$$

Correction to one meson exchange interaction are obtained from the odd powers in the expansion of U_5 .

$$\begin{aligned} (f_{I1}(F_\pi))_{11} &= \left(1 - \frac{5 F_\pi}{3 \cdot 3!} + \frac{35 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\pi^0}{f_\pi} \\ &\quad + \left(\frac{9 F_\pi}{3 \cdot 3!} + \frac{75 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\eta}{\sqrt{3}} \\ (f_{I1}(F_\pi))_{22} &= - \left(1 - \frac{5 F_\pi}{3 \cdot 3!} + \frac{35 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\pi^0}{f_\pi} \\ &\quad + \left(\frac{9 F_\pi}{3 \cdot 3!} + \frac{75 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\eta}{\sqrt{3}} \\ (f_{I1}(F_\pi))_{12} &= \left(1 - \frac{5 F_\pi}{3 \cdot 3!} + \frac{35 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\sqrt{2} \pi^+}{f_\pi} \\ (f_{I1}(F_\pi))_{21} &= \left(1 - \frac{5 F_\pi}{3 \cdot 3!} + \frac{35 F_\pi^2}{9 \cdot 5!} \dots\right) \frac{\sqrt{2} \pi^-}{f_\pi} \\ (f_{I1}(F_\pi))_{13} &= \left(1 - \frac{F_\pi}{3!} + \frac{5 F_\pi^2}{3 \cdot 5!} \dots\right) \frac{\sqrt{2} k^+}{f_\pi} \\ (f_{I1}(F_\pi))_{31} &= \left(1 - \frac{F_\pi}{3!} + \frac{5 F_\pi^2}{3 \cdot 5!} \dots\right) \frac{\sqrt{2} k^-}{f_\pi} \\ (f_{I1}(F_\pi))_{23} &= \left(1 - \frac{F_\pi}{3!} + \frac{5 F_\pi^2}{3 \cdot 5!} \dots\right) \frac{\sqrt{2} k^0}{f_\pi} \\ (f_{I1}(F_\pi))_{32} &= \left(1 - \frac{F_\pi}{3!} + \frac{5 F_\pi^2}{3 \cdot 5!} \dots\right) \frac{\sqrt{2} \bar{k}^0}{f_\pi} \\ (f_{I1}(F_\pi))_{33} &= 0 \end{aligned} \quad (\text{A29})$$

The correction to two meson exchange terms come from the even terms of U_5 .

$$\begin{aligned} (f_{I2})_{11} &= \left(1 - \frac{5 F_\pi}{18} + \frac{7 F_\pi^2}{216}\right) \pi^+ \pi^- + \left(1 - \frac{5 F_\pi}{18} + \frac{7 F_\pi^2}{216}\right) \pi^{02} \\ &\quad + \left(1 - \frac{7 F_\pi}{36} + \frac{11 F_\pi^2}{648}\right) K^+ K^- \\ &\quad + \left(1 - \frac{F_\pi}{12} + \frac{F_\pi^2}{81}\right) \eta^2 \end{aligned}$$

$$\begin{aligned}
(f_{I2})_{22} &= (1 - \frac{5 F_\pi}{18} + \frac{7 F_\pi^2}{216})\pi^+\pi^- + (1 - \frac{5 F_\pi}{18} + \frac{7 F_\pi^2}{216})\pi^0{}^2 \\
&\quad + (1 - \frac{7 F_\pi}{36} + \frac{11 F_\pi^2}{648})K^0\bar{K}^0 \\
&\quad + (1 - \frac{F_\pi}{12} + \frac{F_\pi^2}{81})\eta^2 \\
(f_{I2})_{33} &= (1 - \frac{2 F_\pi}{4!} + \frac{10 F_\pi^2}{3 \cdot 6!})K^+K^- + (1 - \frac{2 F_\pi}{4!} + \frac{10 F_\pi^2}{3 \cdot 6!})K^0\bar{K}^0
\end{aligned} \tag{A30}$$

The correction to meson masses comes through the correction to kinetic energy term.

$$\begin{aligned}
f_{2\pi} &= 1 - \frac{F_\pi}{9} + \frac{4 F_\pi^2}{27 \cdot 3!} \dots\dots \\
f_{2k} &= 1 - \frac{7 F_\pi}{12} + \frac{11 F_\pi^2}{108} \dots\dots \\
f_{2\eta} &= 0
\end{aligned} \tag{A31}$$

The $f_{2\eta} = 0$ implies that there is no change in the mass of η meson in the medium.

Case b: Three flavour symmetric matter

Three flavour symmetric matter consist of a single eight component meson field ϕ . Also all the quark masses become equal. The corrections are obtained by putting $j_\pi = j_K = j_\eta = 8$. The correction to mass term is given by,

$$f_1(F_\phi) = 1 - \frac{2F_\phi}{3 \cdot 2!} + \frac{5 F_\phi^2}{6 \cdot 4!} - \frac{95 f_\phi^3}{72 \cdot 6!} \dots\dots\dots \tag{A32}$$

The one meson exchange term is given by,

$$f_{I1}(F_\phi) = (1 - \frac{10 F_\phi}{8 \cdot 3!} + \frac{380 F_\phi^2}{3 \cdot 8^2 \cdot 5!} \dots\dots) \frac{\Phi}{f_\pi} \tag{A33}$$

The two meson exchange term is

$$f_{I2} = (1 - \frac{313 F_\phi}{576} + \frac{4859 F_\phi^2}{103680})\phi^2 \tag{A34}$$

The correction to mass term is given by,

$$f_2(F_\phi) = 1 - \frac{F_\phi}{8} + \frac{463 F_\phi^2}{1620 \cdot 8^2} \dots\dots\dots \tag{A35}$$

Case c: Two flavour matter

In case of two flavour matter, only pions are present in the medium. So that the meson matrix is now a 2×2 matrix. The contraction rules are

as given for pions in case (a). The results are same as the one obtained for the pions in case (a). All the corrections are now summable and take the forms as given below.

$$f_1 = (1 - F_\pi)e^{-F/6} \quad (\text{A36})$$

$$f_2 = \frac{1}{3}[2 + \frac{1.5}{F_\pi}(1 - e^{-2F/3})] \quad (\text{A37})$$

$$f_{I1} = (1 - \frac{F_\pi}{9})e^{-F_\pi/6} \quad (\text{A38})$$

where the eqn. (A36), eqn. (A37) and eqn. (A38) are the corrections to quark mass term, pion mass term and quark- pion interaction term.

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FIGURES

FIG. 1. Baryon density vs. F_π and quark mass m_q (u and d) plot **(a)** and **(b)** correspond to F_π and m_q for flavour asymmetric case; **(c)** and **(d)** correspond to F_π and m_q for flavour symmetric case respectively.

FIG. 2. Temperature vs. F_π and quark mass m_q (u and d) plot **(a)** and **(b)** correspond to F_π and m_q for flavour asymmetric case; **(c)** and **(d)** correspond to F_π and m_q for flavour symmetric case respectively.

FIG. 3. Baryon density vs. m_q for different pion mass m_π , **(a)** for $m_\pi = 110MeV$, **(b)** for $m_\pi = 90MeV$ and **(c)** for $m_\pi = 70MeV$.

FIG. 4. Baryon density vs. Pressure for different temperatures, **(a)** $T = 0MeV$, **(b)** $T = 50MeV$, **(c)** $T = 80MeV$ and **(d)** $T = 110MeV$.

FIG. 5. Pressure and energy variation with temperature above critical temperature for zero chemical potential, **(a)** $3P/T^4$ and **(b)** E/T^4